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Optimal Resource Allocation for Wireless Mesh Networks

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6.1 Introduction

Wireless networks enable ubiquitous information and computational resource access, and become a popular networking solution. Recently, wireless mesh networks (WMN) [1, 2, 3, 4, 5, 6, 7] have attracted increasing attention and deployment as a high-performance and low-cost solution to last-mile broadband Internet access.

In this chapter, we study the problem of *resource allocation* in wireless mesh networks. Our goal is to design effective resource allocation algorithms for wireless mesh networks, which are *optimal* with respect to resource utilization and *fair* across different network access points. Compared with traditional wireline networks, the unique characteristics of wireless mesh networks pose great challenges to such algorithms. Particularly, the wireless interference issue of mesh networks needs fresh treatment: flows not only contend at the same wireless mesh router (contention in the time domain), but also compete for the shared channel if they are within the interference ranges of each other (contention in the spatial domain). This challenge calls for a new resource allocation framework that could characterize the unique features of wireless mesh networks.

To address this challenge, we present a price-based resource allocation framework for wireless mesh networks to achieve *optimal* resource utilization and *fairness* among competing aggregated flows. In this chapter, we first model the resource allocation problem as an optimization problem: given network resources with constrained capacities and a set of users (*e.g.*, aggregated flows from access points of mesh networks), one tries to allocate resources to each user in a way that the overall satisfaction (so called *utility*) of all users are maximized. We show that such an optimization goal could naturally lead to different fairness objectives when appropriate utility functions are specified. We further present a price-based distributed algorithm which solves this optimization problem and thus provides *fair* and *optimal* resource allocation.

We instantiate the above generalized resource allocation framework to the wireless mesh networks. The key challenge comes from the *shared-medium multi-hop* nature of such networks, namely location-dependent contention and spatial reuse.

Based on solid theoretical analysis, we show that a resource element in a multihop wireless mesh network is a *facet of the polytope defined by the independent set of the conflict graph of this network, which could be approximated by a maximal clique*. Thus we build our price-based resource allocation framework on the notion of *maximal cliques* in wireless mesh networks, as compared to individual links in traditional wide-area wireline networks. We further present a price-based distributed algorithm, which is proven to converge to the global network optimum with respect to resource allocation. The algorithm is validated and evaluated through simulation study.

Our theoretical resource allocation framework of wireless mesh networks possesses great practical advantages. First, with the evolution of wireless signaling technology, medium access and routing protocols, the solution space of this problem may keep reforming, but its nature of optimal resource allocation remains unchanged. A good theoretical framework can effectively decouple the "core" of the problem and its other components (e.g., definition of network resource, and the way it is assigned to users), so that the basic problem formulation and its solution methodology survive. Second, perfect solutions often do not exist, since finding the optimal resource allocation (optimal point in the solution space) is always extremely expensive, if not impossible. When one designs practical solutions to approximate this optimal point, the role of a theoretical framework becomes crucial as it provides philosophical guidance of what is a good intuition.

The rest of this paper is organized as follows. Section 6.2 introduces the generalized resource allocation framework. Section 6.3 instantiates this framework to the case of wireless mesh network. Section 6.4 presents the price-based decentralized resource allocation algorithm. Finally, we show simulation results in Section 6.5, discuss related works in Section 6.6 and conclude in Section 6.7.

6.2 Price-based Resource Allocation Theoretical Framework

In this section, we present the generalized price-based theoretical framework for resource allocation in the setting of an abstract network model. We first formulate the resource allocation problem as an optimization problem. We then show that a price-based approach can provide a decentralized algorithm to solve this problem.

6.2.1 Resource Allocation: An Optimization Problem

An abstract network model

In our abstract network model, a network is represented as a set of *resource elements* E. A resource element $e \in E$ can be a wireline link, a shared wireless channel, etc. Each element has a fixed and finite capacity C_e. Note that the most important nature of a resource element is the independence of its capacity. Specifically, how resources are allocated can not affect the capacity of a resource element. In this sense, a wireline link is a resource element, while a wireless link is not, as its capacity may vary depending on the traffic in its neighborhood and the scheduling algorithm in use.

Characterizing the resource elements in a wireless mesh network is an important yet difficult issue, which will be elaborated in Section 6.3.

This network is shared by a set of flows (*e.g.*, end-to-end aggregated flows in mesh network) F. A flow $f \in F$ has a rate of x_f. f must traverse a sequence of resource elements (*i.e.*, the end-to-end path of f passes multiple links) to reach its destination. Let R_{ef} be the amount of resource e used by a unit flow of f, and y_e be the amount of traffic generated by all flows in F through resource element e. Obviously $y_e = \sum_{f \in F} R_{ef} x_f$. Note that the calculation of R_{ef} depends on the definition of resource element, which may vary for different types of networks.

Objective: maximizing aggregated utility

We associate each end-to-end flow $f \in F$ with a *utility function* $U_f(x_f) : \Re_+ \rightarrow \Re_+$, which represents the degree of satisfaction of its associated end user. Here we make the following assumptions about $U_f(x_f)$:

– **A1**. On the interval $[0, \infty)$, the utility function $U_f(\cdot)$ is increasing, strictly concave and continuously differentiable.
– **A2**. U_f is additive so that the aggregated utility of rate allocation $\boldsymbol{x} = (x_f, f \in F)$ is $\sum_{f \in F} U_f(x_f)$.

We investigate the problem of optimal resource allocation in the sense of *maximizing the aggregated utility function* of all users, which is also referred to as the *social welfare* in the literature. Formally, this objective is given as follows,

$$\textbf{maximize} \sum_{f \in F} U_f(x_f)$$

This optimization objective is of particular interest. As we will demonstrate shortly, such an objective achieves Pareto optimality with respect to the resource utilization, and also realizes different fairness models — including proportional and max-min fairness — when appropriate utility functions are specified.

Constraint: resource element and its capacity

Recall that each element $e \in E$ in the network has a finite capacity C_e, and y_e is the amount of traffic generated by all flows in F through resource element e. The constraints on resource capacities are given as follows,

$$\forall e \in E, y_e \leq C_e$$

As R_{ef} is the amount of resource e used by a unit flow of f, we have $y_e = \sum_{f \in F} R_{ef} \cdot x_f$. Thus the resource constraint is given as follows.

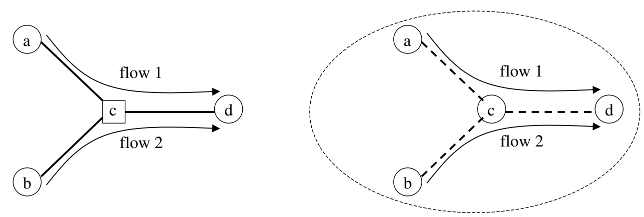
$$\forall e \in E, \sum_{f \in F} R_{ef} \cdot x_f \leq C_e$$

or in concise form,

$$\boldsymbol{R} \cdot \boldsymbol{x} \leq \boldsymbol{C}$$

where $\boldsymbol{R} = (R_{ef})_{|E|\times|F|}$ is a matrix with element R_{ef} at row e and column f, and $\boldsymbol{x} = (x_f, f \in F)$, $\boldsymbol{C} = (C_e, e \in E)$ are vectors of flow rates and resource capacities respectively.

The definition of resource element and its capacity can vary for different types of networks. It is particularly hard to define for wireless mesh networks. In what follows, we will illustrate the concept of resource element in two simple network settings and present their resource constraints. Based on these intuitive examples, we will further define the resource model of a wireless mesh network in Section 6.3, which establishes the foundation of theoretical study on resource allocation in this type of network.



(a) Wireline network (b) Wireless network without spatial reuse

Fig. 6.1. Resource elements in different networks.

Wireline networks

In wireline networks, flows only contend with each other if they share the same physical link. In this case, the resource element e is a wireline link, its resource capacity C_e is the link capacity. In this case, $\boldsymbol{R}$ can be understood as the routing matrix defined as follows.

$$R_{ef} = \begin{cases} 1 & \text{if } f \text{passes through } e \\ 0 & \text{otherwise} \end{cases}$$

In the example shown in Fig. 6.1 (a), the constraints on resource allocations of flows 1 and 2 can be expressed as

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} C_{ac} \\ C_{bc} \\ C_{cd} \end{pmatrix}$$

Wireless networks without spatial reuse

We now consider a simple wireless network based on unit disk graph model as shown in Fig. 6.1 (b). All four nodes are within the transmission range of each other and have the same data transmission rate. Flow 1 and 2 not only contend at the wireless link $\{c, d\}$ which they both traverse, but also at the link $\{a, c\}$ and $\{b, c\}$

which share the same wireless channel. Hence in this case, the wireless channel shared by these three links is the only resource element, whose capacity is C_{chan} – the wireless channel capacity. Since each flow passes two hops in this wireless channel, thus $R_{ef_1} = R_{ef_2} = 2$. Then the constraint on resource allocation of flow 1 and 2 can be expressed as

$$\begin{pmatrix} 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq C_{chan}$$

Putting things together

Summarizing above discussions, we formulate the resource allocation problem in a generalized form as follows.

$$\mathbf{S} : \textbf{maximize} \sum_{f \in F} U_f(x_f) \quad (6.1)$$

$$\textbf{subject to } \boldsymbol{R} \cdot \boldsymbol{x} \leq \boldsymbol{C} \quad (6.2)$$

$$\boldsymbol{x} \geq \mathbf{0} \quad (6.3)$$

The objective function in Eq. (6.1) maximizes the aggregated utility of all flows. The constraint of the optimization problem (Inequality (6.2)) comes from the resource constraint of the network. We now demonstrate that, by optimizing towards such an objective, both *optimal resource utilization* and *fair resource allocation* may be achieved among *end-to-end* flows.

Pareto optimality

With respect to optimal resource utilization, we show that the resource allocation is *Pareto optimal* if the optimization problem $\mathbf{S}$ can be solved. Formally, Pareto optimality is defined as follows.

Definition 1. (Pareto optimality) *A rate allocation* $\boldsymbol{x} = (x_f, f \in F)$ *is* **Pareto optimal***, if it satisfies the following two conditions: (1)* $\boldsymbol{x}$ *is feasible,* i.e., $\boldsymbol{x} \geq \mathbf{0}$ *and* $\boldsymbol{R} \cdot \boldsymbol{x} \leq \boldsymbol{C}$*; and (2)* $\forall \boldsymbol{x}'$ *which satisfies* $\boldsymbol{x}' \geq \mathbf{0}$ *and* $\boldsymbol{R} \cdot \boldsymbol{x}' \leq \boldsymbol{C}$*, if* $\boldsymbol{x}' \geq \boldsymbol{x}$*, then* $\boldsymbol{x}' = \boldsymbol{x}$*. In the second condition, The* $\geq$ *relation is defined such that, two vectors* $\boldsymbol{x}$ *and* $\boldsymbol{x}'$ *satisfy* $\boldsymbol{x}' \geq \boldsymbol{x}$*, if and only if for all* $f \in F$*,* $x'_f \geq x_f$*.*

Proposition 1. *A rate allocation* $\boldsymbol{x}$ *is Pareto optimal, if it solves the problem* $\mathbf{S}$*, with increasing utility functions* $U_f(x_f)$*, for* $f \in F$*.*

Proof. Let $\boldsymbol{x}$ be a solution to the problem $\mathbf{S}$. If $\boldsymbol{x}$ is not Pareto optimal, then there exists another vector $\boldsymbol{x}' \neq \boldsymbol{x}$, which satisfies $\boldsymbol{R} \cdot \boldsymbol{x}' \leq \boldsymbol{C}$ and $\boldsymbol{x}' > \boldsymbol{x}$. As $U_f(\cdot)$ is increasing, we have that $\sum_{f \in F} U_f(x'_f) > \sum_{f \in F} U_f(x_f)$. This leads to a contradiction, as $\boldsymbol{x}$ is the solution to $\mathbf{S}$ and hence maximizes $\sum_{f \in F} U_f(x_f)$.

Fairness

By choosing appropriate utility functions, the optimal resource allocation can implement different fairness models among the flows. We illustrate this fact using two commonly adopted fairness models: *weighted proportional* fairness and *max-min* fairness.

Definition 2. (weighted proportional fairness) *A vector of rates* $\boldsymbol{x} = (x_f, f \in F)$ *is* **weighted proportionally fair** *with the vector of weights* w_f*, if it satisfies the following two conditions: (1)* $\boldsymbol{x}$ *is feasible,* i.e., $\boldsymbol{x} \geq 0$ *and* $\boldsymbol{R} \cdot \boldsymbol{x} \leq \boldsymbol{C}$*; and (2) for any other feasible vector* $\boldsymbol{x}' = (x'_f, f \in F)$*, the aggregation of proportional changes is zero or negative:*

$$\sum_{f \in F} w_f \frac{x'_f - x_f}{x_f} \leq 0$$

Proposition 2. *A rate allocation* $\boldsymbol{x}$ *is weighted proportional fair with the weight vector* w_f*, if and only if it solves the problem* $\mathbf{S}$*, with* $U_f(x_f) = w_f \log x_f$ *for* $f \in F$*.*

Proof. As shown in [8], by the optimality condition (6.1), this proposition can be derived according to the following relation:

$$\sum_{f \in F} \frac{\partial U_f}{\partial x'_f}(x_f)(x'_f - x_f) = \sum_{f \in F} w_f \frac{x'_f - x_f}{x_f} < 0$$

where the strict inequality follows from the strict concavity of U_f.

Definition 3. (max-min fairness) *A vector of rates* $\boldsymbol{x} = (x_f, f \in F)$ *is* **max-min fair***, if it satisfies the following two conditions: (1)* $\boldsymbol{x}$ *is feasible,* i.e., $\boldsymbol{x} \geq 0$ *and* $\boldsymbol{R} \cdot \boldsymbol{x} \leq \boldsymbol{C}$*; and (2) for any* $f \in F$*, increasing* x_f *can not be achieved without decreasing the fair share* $x_{f'}$ *of another flow* $f' \in F$ *that satisfies* $x_f \geq x_{f'}$*.*

Proposition 3. *A rate allocation* $\boldsymbol{x}$ *is max-min fair if and only if it solves the problem* $\mathbf{S}$*, with* $U_f(x_f) = -(-\log x_f)^\zeta$*,* $\zeta \rightarrow \infty$ *for* $f \in F$*.*

These results straightforwardly follow their counterparts in wireline networks [8].

6.2.2 Decentralized Solution: A Price-based Approach

We proceed to study the decentralized solution to the problem $\mathbf{S}$ so that the optimal resource allocation can be achieved.

By assumption **A1**, the objective function of $\mathbf{S}$ in Eq. (6.1) is differentiable and strictly concave. In addition, the feasible region of the optimization problem in Inequality (6.2) is convex and compact. By non-linear optimization theory, there exists a unique maximizing value of argument $\boldsymbol{x}$ for the above optimization problem. Let us consider the Lagrangian form of the optimization problem $\mathbf{S}$:

$$L(\boldsymbol{x};\boldsymbol{\mu}) = \sum_{f\in F} U_f(x_f) + \boldsymbol{\mu}^T(\boldsymbol{C} - \boldsymbol{Rx}) \\ = \sum_{f\in F}(U_f(x_f) - x_f \sum_{e\in E}\mu_e R_{ef}) + \sum_{e\in E}\mu_e C_e \tag{6.4}$$

where $\boldsymbol{\mu} = (\mu_e, e \in E)$ is a vector of Lagrange multipliers. Given global knowledge of utility functions, **S** is mathematically tractable. However, in practice, such knowledge is unlikely to be available. In addition, it may be infeasible to compute and allocate resources in a centralized fashion. Here we seek a decentralized solution. The key to decentralization is pricing.

In the Lagrangian form specified in Eq. (6.4), the Lagrange multipliers μ_e may be regarded as the implied cost, or the *shadow price*, of a unit flow using resource e. Such a price μ_e reflects the traffic load y_e at the resource element e. Flow $f \in F$ will then be charged with a flow price λ_f which is the sum of the costs of all resource elements it uses, the cost of each resource element being the product of its price and the amount of resource used by a unit flow of f, namely,

$$\lambda_f = \sum_{e\in E} R_{ef}\mu_e$$

Based on the flow price λ_f, flow f can make a self-optimized decision to adjust its sending rate x_f. The aggregated sending rate $y_e = \sum_{f\in F} R_{ef} \cdot x_f$ of all flows in resource element e in turn affects its price μ_e. To summarize, Fig. 6.2 illustrates the price-based resource allocation framework. Here we deem each component in the diagram as abstract entities capable of computing and communicating. This framework involves no central authority and purely depends on local decision of each component and exchange of control signals among them. In each cycle, a resource element e calculates its load y_e, the total amount of flows passing through it, then derives its penalty μ_e and sends it to all these flows. Meanwhile, a flow f, on receiving prices from all resource elements it traverses, derives its flow price λ_f, then adjusts flow rate x_f. Such a cycle repeats itself, and finally converges to an equilibrium point.

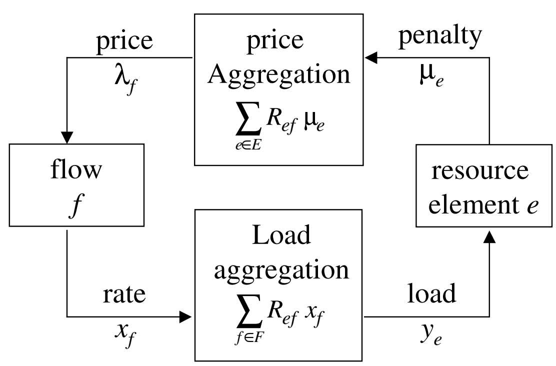


Fig. 6.2. Price-based Resource Allocation Framework.

The presented framework is a generalized form of the framework proposed by Kelly et al. in [9, 8] for wireline networks. As we will show in Section 6.4, such a generalization is critical for us to study the resource allocation problem in wireless mesh networks. We list all notations introduced in Section 6.2 as follows.

Notation	**Definition**
$f \in F$	End-to-end flow in the network
$e \in E$	Resource element of the network
$\boldsymbol{x} = (x_f, f \in F)$	Rate vector of flow $f \in F$
$\boldsymbol{C} = (C_e, e \in E)$	Capacity vector of resource element $e \in E$
$\boldsymbol{R} = (R_{ef})_{\lvert E \rvert \times \lvert F \rvert}$	Resource constraint matrix
$U_f(x_f)$ $(f \in F)$	Utility function of flow $f \in F$
$\boldsymbol{y} = (y_e, e \in E)$	Aggregated traffic load at resource element $e \in E$
$\boldsymbol{\mu} = (\mu_e, e \in E)$	Price of resource element $e \in E$
λ_f $(f \in F)$	Price of flow $f \in F$

Table 6.1. Notations in Section 6.2

6.3 Resource Model of Multihop Wireless Mesh Networks

In this section, we study the resource model and identify the resource elements of a wireless mesh network. We consider a two-tier wireless mesh network shown in Fig. 6.3. In this network, each end host accesses a local access point (LAP). These local access points, along with multiple stationary wireless routers, are also called mesh nodes. These nodes communicate with each other and form a multi-hop wireless backbone. This backbone network eventually forwards user traffic to the gateway access points (GAPs) connected to the Internet via physical wireline connection.

This chapter focuses on the resource allocation issue in wireless mesh backbone network. The goal is to achieve fairness among local access points. In particular, we consider a wireless mesh *backbone* network that consists of a set of nodes N. The transmission of each node $n_i \in N$ follows the unit disk graph model with a transmission range of d_{tx} and an interference range of d_{int}, which can be larger than d_{tx}.

To simplify the discussion, we only consider the scenario where mesh nodes use the same wireless channel. Packet transmission in such a network is subject to location-dependent contention. Here we consider the protocol model proposed in [10]. In this model, the transmission from node n_i to n_j $(n_i, n_j \in N)$ is successful if (1) the distance between these two nodes d_{ij} satisfies $d_{ij} < d_{tx}$, and (2) any node $n_k \in N$, which is within the interference range of the receiving node n_j $(d_{kj} \leq d_{int})$, is not transmitting. This model can be further refined to the case of IEEE 802.11-style MAC protocol, where the sending node n_i is also required to be free of interference as it needs to receive the link layer acknowledgement from the receiving

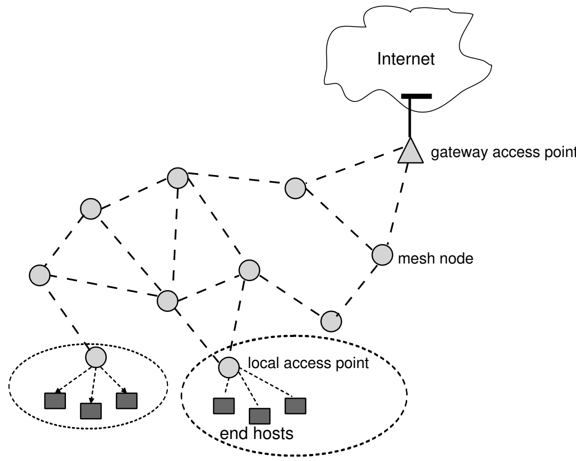


Fig. 6.3. A Wireless Mesh Network Example.

node n_j. Specifically, any node $n_k \in N$, which is within the interference range of n_i or n_j ($d_{kj} \leq d_{int}$ or $d_{ki} \leq d_{int}$), is not transmitting. We model such a network as a directional graph $G = (N, L)$, where $L \subseteq N^2$ denotes the set of wireless links.

Now let us consider a *conflict graph* $G_c = (V_c, L_c)$ of network G [11]. A vertex of the conflict graph $v_i \in V_c$ corresponds to a wireless link in the network $l \in L$. There exists an edge between two vertices if the transmissions along these two wireless links contend with each other according to the above protocol model.

To illustrate these concepts, we show an example in Fig. 6.4. Fig. 6.4 (a) gives the network topology and the traffic used in the example. In this example, the transmission and interference range of a node is 250m and 550m respectively. a and b, c and d, e and f are 250m apart. b and c, d and e are 300m apart. Thus the wireless links $\{a, b\}$ and $\{c, d\}$ contend with each other, also do $\{c, d\}$ and $\{e, f\}$. But $\{a, b\}$ and $\{e, f\}$ can transmit simultaneously. The conflict graph of this wireless network is shown in Fig. 6.4 (b).

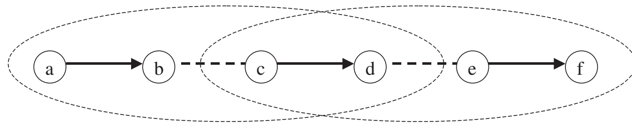
6.3.1 Identifying Resource Elements

Now let us consider an *independent set* $I \subseteq V_c$ of the graph G_c. I can be represented using a $|V_c|$-dimension *independence vector* $\boldsymbol{\iota}_I = (\iota_j, v_j \in V_c)$, defined as follows.

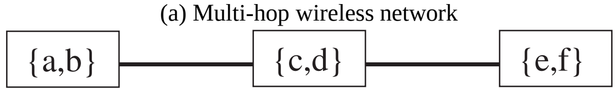
$$\iota_j = \begin{cases} 1 & \text{if } v_j \in I \\ 0 & \text{otherwise} \end{cases}$$

$\boldsymbol{\iota}_I$ can be regarded as a point in a V_c-dimensional *independence space*. In this space, each dimension corresponds to a vertex $v_i \in V_c$.

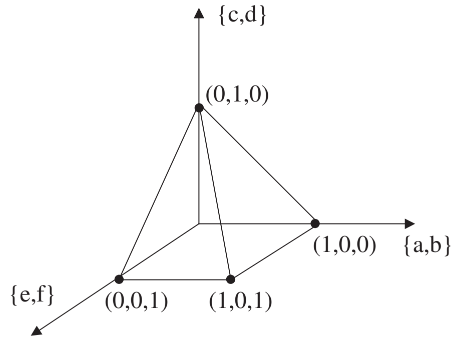
In the above example, besides the independent sets consisting of each vertex itself, $\{\{a, b\}, \{e, f\}\}$ is also an independent set. Let vertices $\{a, b\}$, $\{c, d\}$, $\{e, f\}$



(a) Multi-hop wireless network



(b) Conflict graph



(c) Independent set polytope

Fig. 6.4. Resource model of multi-hop wireless network.

correspond to the three dimensions of the independence space, the following independence vectors are shown in Fig. 6.4 (c): $(1, 0, 0)$,$(0, 1, 0)$,$(0, 0, 1)$ and $(1, 0, 1)$. A special independence vector is the origin point $(0, 0, 0)$.

The picture also shows that a polytope is formed as the convex hull of all points corresponding to each independence vector, or in other words, convex combination of all independence vectors. We call such a polytope the *independent set polytope*, denoted as T_G. Let us consider a $|V_c|$-dimension vector $\boldsymbol{q} = (q_j, v_j \in V_c)$, where q_j is the fraction of time during which link l corresponding to v_j is active. Vector $\boldsymbol{q}$ is *schedulable* if there exists a collision-free MAC transmission schedule that allocates q_j to link l which corresponds to v_j. The result of [11] shows that

Proposition 4. *Vector* $\boldsymbol{q} = (q_j, v_j \in V_c)$ *is schedulable if and only if it lies within the independent set polytope* T_G.

Reflected in Fig. 6.4 (c), all points within the polytope T_G is schedulable. To model the resource element from this concept, we consider the *facets* of the polytope T_G[1]. Note that we can get these facets by running any polynomial-time convex hull algorithm[12] on all vertices of the polytope (independence vectors). We collect all facets into a set $\boldsymbol{\Phi}$. The plane that a facet $\boldsymbol{\phi}_i \in \boldsymbol{\Phi}$ belongs to can be presented in the following linear form.

[1] The facets that lie along the coordination plane are excluded.

$$\sum_{j=1}^{|V_c|} \phi_{ij} q_j - Z_i = 0$$

where ϕ_{ij} and Z_i are coefficients of the plane function. If we formulate $\boldsymbol{\Phi}$ into a matrix $\boldsymbol{\Phi} = (\phi_{ij})_{|\boldsymbol{\Phi}| \times |V_c|}$, then the polytope T_G can be represented in the following vector form.

$$\boldsymbol{\Phi} \cdot \boldsymbol{q} \leq \boldsymbol{Z}$$

where $\boldsymbol{Z} = (Z_i, \boldsymbol{\phi}_i \in \boldsymbol{\Phi})$.

When the data rates of all wireless links are the same, we call such a rate the capacity of the wireless channel and denote it as C_{chan}. If we scale the independent set polytope T_G by C_{chan}, then under the ideal centralized MAC scheduling, this polytope represents the solution space of our problem. In other words, a *wireless link rate allocation* $\boldsymbol{y} = (y_l, l \in L)$ is feasible, if the following condition holds.

$$\boldsymbol{\Phi} \cdot \boldsymbol{y} \leq C_{chan} \cdot \boldsymbol{Z} \tag{6.5}$$

Let $\boldsymbol{A} = (A_{lf})_{|L| \times |F|}$ be the routing matrix defined as follows,

$$A_{lf} = \begin{cases} 1 & \text{if flow } f \text{ passes through wireless link } l \\ 0 & \text{otherwise} \end{cases}$$

then it is easy to see that $\boldsymbol{A} \cdot \boldsymbol{x} = \boldsymbol{y}$. Substituting Inequality (6.5) into the constraint of problem **S** in (6.2), we can easily derive that $\boldsymbol{R} = \boldsymbol{\Phi} \cdot \boldsymbol{A}$ and $\boldsymbol{C} = C_{chan} \cdot \boldsymbol{Z}$. From these properties, we observe that *each facet* $\phi_i \in \boldsymbol{\Phi}$ *can be regarded as a resource element with an independent capacity* $C_{chan} \cdot Z_i$. Though the concept of independent set has been used in the existing works to explore the throughput limit of multihop wireless networks [14], the concept of facets of independent set polytope is never discussed. However, constructing the solution space by the formulation of facets is critical to the development of a decentralized algorithm.

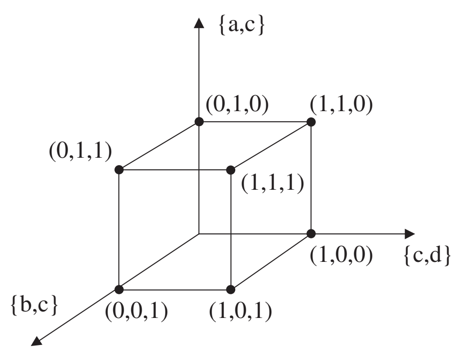


Fig. 6.5. Solution space of wireline network in Fig. 6.1 (a)

As an interesting finding, we also notice that the wireline network model is a special case of this formulation. Take the network shown in Fig. 6.4 (a) as an example, since all links are independent from each other, any subset of the link set L is an independent set. The resulting independent set polytope (with each dimension normalized by the capacity of its corresponding link) is shown in Fig. 6.5. An important property of this polytope is that it is a *cube*. That means for each of its facets, the plane it belongs to only intersects with one axis. Since each axis represents a wireline link, back to Inequality (6.5), this implies that (1) $\boldsymbol{\Phi} = \boldsymbol{I}$, the identity matrix, thus $\boldsymbol{A} = \boldsymbol{R}$, and (2) $Z_i = 1$ for any $\boldsymbol{\phi}_i \in \boldsymbol{\Phi}$.

Note that the above formulation could be easily extended to the case of heterogeneous wireless link data rates. We denote the wireless link data rates using a vector $\boldsymbol{b} = (b_l, l \in L)$. It is obvious that $q_j = \frac{y_l}{b_l}$ for v_j which corresponds to l. Let $\boldsymbol{b}' = (1/b_l, l \in L)$. It is obvious that $\boldsymbol{q} = \boldsymbol{y} \cdot \boldsymbol{b}'^T$. Thus the constraint for wireless link rate allocation is give as follows.

$$\boldsymbol{\Phi} \cdot \boldsymbol{y} \cdot \boldsymbol{b}'^T \leq \boldsymbol{Z}$$

For simplicity, we only consider the homogeneous wireless link rate in the following discussions.

6.3.2 Approximating Resource Element

To this end, we have clearly identified the resource elements of a multihop wireless mesh backbone network. However, applying this model to the resource allocation framework can still be difficult, as the problem of finding all independent sets is NP-hard. Besides, this model assumes ideal MAC scheduling. It is difficult to be applied for practical implementation in realistic wireless network settings, (e.g., with non-ideal MAC algorithm such as IEEE 802.11), because the facets of the independent set polytope of the contention graph lack the intuition to be mapped into any instance in the physical wireless network, not to mention to be implemented via distributed algorithms.

To address this difficulty, we explore the approximation of resource elements by studying the upper bound of the resource constraint in a multihop wireless network. Here we present a *maximal-clique-based* approximation. Such an approximation gives a good intuitive explanation on the structure of the resource element in the physical network. Thus it can also facilitate the distributed implementation of resource allocation algorithms.

In a graph, a complete subgraph is referred to as a *clique*. A *maximal clique* is defined as a clique that is not contained in any other cliques[2]. In a contention graph, the vertices in a maximal clique represent a maximal set of mutually contending wireless links, along which at most one subflow may transmit at any given time. Intuitively, each maximal clique in a contention graph represents a maximal distinct contention region, since at most one subflow in the clique can transmit at any time, and adding

[2] Note that the *maximal clique* has a different definition from the *maximum clique* of a graph, which is the maximal clique with the largest number of vertices. Finding the maximum clique of a graph is a NP-complete problem, while enumerating all the maximal cliques of a graph can be solved in polynomial time [13].

any other flows into this clique will introduce the possibility of simultaneous transmissions. We denote the set of all maximal cliques in G_c as Q.

Based on the above discussions, we have the following results for rate allocation vector $\boldsymbol{y}$:

Proposition 4. *Rate allocation $\boldsymbol{y} = (y_l, l \in L)$ is feasible, then the following condition is satisfied.*

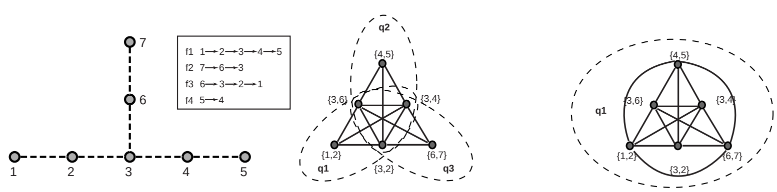
$$\forall e \in Q, \sum_{l \in V(e)} y_l \leq C_{chan} \tag{6.6}$$

where $V(e) \subseteq L$ is the set of vertices in clique e.

Eq. (6.6) gives an upper bound on the rate allocations to the wireless links. Such a bound may not be tight. First, there may exist no schedules that assign rates to the wireless links to achieve this bound. Such scenario happens when the contention graph has odd holes or odd anti-holes [14]. Second, for some contention graphs, even if there exists an ideal centralized scheduling algorithm that can achieve this bound, the distributed scheduling algorithms that employ carrier sensing multiple access (*e.g.*, IEEE 802.11) can not achieve this bound. To address the above issues, we introduce C_e, the *achievable* channel capacity at a clique e so that if $\sum_{l \in V(e)} y_l \leq C_e$ then $\boldsymbol{y} = (y_l, l \in L)$ is feasible. To this end, we observe that a maximal clique e can be regarded as an approximation of a resource element with capacity C_e.

We now proceed to consider the resource constraint of a wireless mesh backbone network using the maximal clique as the approximation of the resource element. In particular, we define a clique-flow matrix $\boldsymbol{R} = \{R_{ef}\}$, where $R_{ef} = |V(e) \cap L(f)|$ represents the number of subflows that flow f has in the clique e. If we treat a maximal clique as a resource element, then the clique-flow matrix $\boldsymbol{R}$ represents the "resource usage pattern" of each flow. Let the vector $\boldsymbol{C} = (C_e,\ e \in Q)$ be the vector of achievable channel capacities in each of the cliques. Constraints with respect to rate allocations to end-to-end aggregated flows are presented in the following proposition.

Proposition 5. *In a multi-hop wireless network $G = (N, L)$ with a set of flows F, there exists a feasible rate allocation $\boldsymbol{x} = (x_f, f \in F)$, if and only if $\boldsymbol{R} \cdot \boldsymbol{x} \leq \boldsymbol{C}$.*



(a) Network topology (b) Wireless link contention graph ($d_{tx} = d_{int}$) (c) Wireless link contention graph ($2d_{tx} = d_{int}$)

Fig. 6.6. Approximate resource model of multihop wireless network.

We present an example to illustrate the above concepts and notations. Fig. 6.6(a) shows the topology of the network, as well as its ongoing flows. The corresponding contention graph is shown in Fig. 6.6(b). In this example, there are 4 end-to-end flows $f_1 = \{\{1,2\},\{2,3\},\{3,4\},\{4,5\}\}$, $f_2 = \{\{7,6\},\{6,3\}\}$, $f_3 = \{\{6,3\},\{3,2\},\{2,1\}\}$ and $f_4 = \{\{5,4\}\}$. As such, in Fig. 6.6(b) there are three maximal cliques in the contention graph: $e_1 = \{\{1,2\},\{3,2\},\{3,4\},\{3,6\}\}$, $e_2 = \{\{3,2\},\{3,4\},\{4,5\},\{3,6\}\}$ and $e_3 = \{\{3,2\},\{3,4\},\{3,6\},\{6,7\}\}$.

We use y_{ij} to denote the aggregated rate of *all* subflows along wireless link $\{i,j\}$. For example, $y_{12} = x_1 + x_3$, $y_{36} = x_2 + x_3$. In each clique, the aggregated rate may not exceed the corresponding channel capacity. That is

$$y_{12} + y_{32} + y_{34} + y_{36} \le C_1 \tag{6.7}$$

$$y_{32} + y_{34} + y_{45} + y_{36} \le C_2 \tag{6.8}$$

$$y_{32} + y_{34} + y_{36} + y_{67} \le C_3 \tag{6.9}$$

When it comes to end-to-end flow rate allocation, the resource constraint imposed by shared wireless channels is as follows.

$$\begin{pmatrix} 3 & 1 & 3 & 0 \\ 3 & 1 & 2 & 1 \\ 2 & 2 & 2 & 0 \end{pmatrix} \cdot \boldsymbol{x} \le \boldsymbol{C}$$

We collect notations introduced in this section into Tab. 6.2.

Notation	**Definition**
$n_i \in N$	Wireless node
$l = \{n_i, n_j\} \in L$	Wireless link connecting nodes n_i and n_j
d_{ij}	Distance between nodes n_i and n_j
$G = (N, L)$	Wireless network
$G_c = (N_c = L, L_c)$	Conflict graph of G
$I \subseteq V_c$	Independent set of G_c
$\boldsymbol{\iota}_I = (\iota_j, v_j \in V_c)$	Independence vector of I
T_G	Independent set polytope of G_c
$\boldsymbol{q} = (q_j, v_j \in V_c)$	Active time scheduling vector for all links
$\boldsymbol{y} = (y_l, l \in L)$	Link flow scheduling vector
$\boldsymbol{\Phi} = (\phi_{ij})_{\lvert\boldsymbol{\Phi}\rvert \times \lvert V_c\rvert}$	Facet matrix of polytope T_G
$\boldsymbol{Z} = (Z_i, \boldsymbol{\phi}_i \in \boldsymbol{\Phi})$	Facet coefficient vector of polytope T_G
C_{chan}	Wireless channel capacity
$\boldsymbol{A} = (R_{lf})_{\lvert L\rvert \times \lvert F\rvert}$	Routing matrix

Table 6.2. Notations in Sec. 6.3

6.4 Price-based Resource Allocation Algorithm

We now present the decentralized algorithms for resource allocation in multihop wireless networks based on the theoretical framework in Section 6.2 and the resource model in Section 6.3.

6.4.1 Price Model

We first illustrate the concepts and components of the generalized resource allocation framework in the setting of wireless mesh network. Recall that a resource element e in multihop wireless networks is a set of wireless links defined by a facet of independent set polytope $\boldsymbol{\phi}_i \in \boldsymbol{\Phi}$, which could be approximated by a maximal clique $e \in Q$. Thus the amount of traffic y_e at the resource element e is the sum of traffic at the wireless links that belong to the resource element e:

$$y_e = \sum_{l \in e} y_l$$

As a wireless link l may be the member of several resource elements e, we define the price of wireless link as follows

$$\mu_l = \sum_{e: l \in e} \mu_e$$

Thus the price of a flow f can be represented in two alternative ways.

$$\lambda_f = \sum_{e \in E} R_{ef} \mu_e \tag{6.10}$$

$$= \sum_{l: f \text{ passes } l} \mu_l \tag{6.11}$$

The first representation (6.10) can be explained as follows. Flow f needs to pay for all the resource elements it uses. For each resource element, the cost is the product of the number of wireless links that f traverses in this resource element and its price. In the second representation (6.11), flow price is the aggregated price of all wireless links it passes. Note that for each wireless link, its price is the aggregated price of all the resource elements that it belongs to.

6.4.2 Price-based Rate Limiting Algorithm

Assume that resource element prices $\boldsymbol{\mu} = (\mu_e, e \in E)$ are generated appropriately as a function of the load $\boldsymbol{y} = (y_e, e \in E)$ at these resource elements. We first study how flows adjust their resource usages.

As presented in the theoretical framework in Section 6.2, flow f attempts to maximize its net benefit.

$$\max_{x_f}\{U_f(x_f) - \lambda_f \cdot x_f\}$$

A simple first-order condition establishes that

$$U'_f(x_f) = \lambda_f$$

Thus it adapts its rates to equalize the flow price, *i.e.*, $\lambda_f = \sum_{e \in E} \mu_e R_{ef}$, with a target value $U'_f(x_f)$. Formally, the rate adaptation algorithm of flow f can be represented in the following differential equation:

$$\frac{d}{dt} x_f(t) = \gamma \left(1 - \frac{1}{U'_f(x_f(t))} \sum_{e \in E} \mu_e(t) R_{ef}\right) \quad (6.12)$$

where $x_f(t)$ is the rate of f at time t. The price $\mu_e(t) = \mu_e(y_e(t))$ is a non-negative, continuous and increasing function of the total traffic $y_e = \sum_{f \in F} R_{ef} x_f$ at resource e at time t. γ is the amount of adjustment. Alternatively, it can be represented in the discrete time form:

$$x_f[t+1] = x_f[t] + \gamma \left(1 - \frac{1}{U'_f(x_f[t])} \sum_{e \in E} \mu_e[t] R_{ef}\right) \quad (6.13)$$

We now establish the stability of this algorithm. Further we show that, at equilibrium each flow maximizes its own net benefit; moreover the flows collectively solve the relaxation of the original problem. This result is formally presented in the following theorem.

Theorem 1. *Let*

$$\mathcal{V}(\boldsymbol{x}) = \sum_{f \in F} U_f(x_f) - \sum_{e \in E} \int_0^{y_e} \mu_e(z) R_{ef} dz$$

$\mathcal{V}(\boldsymbol{x})$ is a strictly concave function. Moreover, it is a Lyapunov function for the system of the differential equation Eq. (6.12). The unique value $\boldsymbol{x}$ that maximizes $\mathcal{V}(\boldsymbol{x})$ is a stable point of the system, to which all trajectories converge. In addition, $\boldsymbol{x}$ is the unique equilibrium of the discrete time system specified by Eq. (6.13).

Proof. Observe that

$$\frac{\partial \mathcal{V}(\boldsymbol{x})}{\partial x_f} = U'_f(x_f) - \sum_{e \in E} \mu_e(y_e) R_{ef} \quad (6.14)$$

Setting these derivatives to zero identifies the maximum. Further

$$\frac{d\mathcal{V}(\boldsymbol{x}(t))}{dt} = \sum_{f \in F} \frac{\partial \mathcal{V}(\boldsymbol{x})}{\partial x_f} \cdot \frac{dx_f(t)}{dt} \quad (6.15)$$

$$= \gamma \sum_{f \in F} U'_f(x_f) \left(1 - \frac{1}{U'_f(x_f)} \mu_e(y_e) R_{ef}\right)^2 \quad (6.16)$$

establishes that $\mathcal{V}(\boldsymbol{x}(t))$ is strictly increasing with t, unless $\boldsymbol{x}(t) = \boldsymbol{x}^*$, the unique $\boldsymbol{x}$ maximizing $\mathcal{V}(\boldsymbol{x})$. The function $\mathcal{V}(x)$ is thus a Lyapunov function for the system Eq. (6.12), which establishes the result of Theorem 1.

The presented rate adaptation algorithm can be implemented as an ingress rate limiting mechanism for aggregated flows at local access points as in [15].

6.4.3 Discussion

The clique-based resource element definition is based on the assumption of ideal MAC scheduling. Such a definition helps us to define the upper bound of the solution space of the resource allocation problem. Yet, in practice, MAC algorithms, such as IEEE 802.11, can perform much worse than the ideal one. Also in order to calculate the price of a clique, the mesh routers need to communicate with each other to exchange their load, which may incur unnecessary overhead [16]. In recognition of the hardness of this problem, here we present a heuristic algorithm for price generation in IEEE 802.11-based networks. The main propose of this algorithm is to illustrate how the presented theoretical framework can be applied to real network settings.

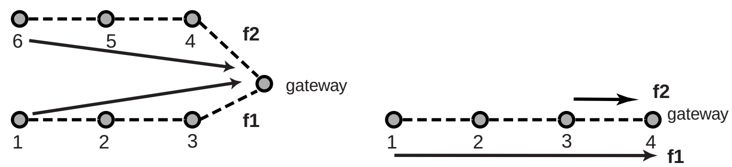
Our approximation is based on two observations. First, due to the characteristics of conflict graph, the wireless links within a clique are most likely to be close to each other geographically. Second, contention window size in IEEE 802.11 can give important hints for traffic load in the neighborhood. Based on these observations, we make the following approximation in the calculation. First, we consider a resource element e consists of a wireless link l and its neighborhood wireless links l' which has one node that is within the range of virtual carrier sense. *i.e.*, one node of l' can hear the RTS or CTS sent from wireless link l. Second, we use the contention window sizes cw of the nodes connecting these links to infer the traffic at this resource element. Formally the price μ_e of resource element e is generated as follows.

$$\mu_e(t) = \beta \cdot m(c\bar{w}_e(t), c\tilde{w}) \tag{6.17}$$

In Eq. (6.17), β is a scaling factor. $c\bar{w}_e(t)$ is the average contention window size within resource element e at time t. Nodes exchange the information of their contention window sizes via piggybacking them onto RTS/CTS control frames. $c\tilde{w}$ is the target contention window size, which is a tunable parameter of this implementation. An ideal target contention window size needs to be tuned according the node density of the network. Function $m(c\bar{w}_e(t), c\tilde{w})$ is defined as the probability that $c\bar{w}_e(t)$ is larger than $c\tilde{w}$. It is easy to see that $m(c\bar{w}_e(t), c\tilde{w})$ is an increasing function of $c\bar{w}_e$ and a decreasing function of $c\tilde{w}$.

6.5 Performance Evaluation

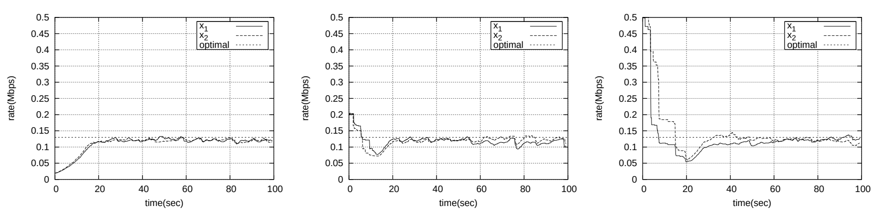
In this section, we evaluate the performance of our resource allocation algorithm. We implement the algorithm based on the wireless extensions in ns-2. In the simulation, the physical wireless channel capacity is 1 Mbps; the utility function is $U_f(x_f) = \log(x_f)$; the packet size is 1000 bytes; the transmission range and interference range are $250m$ and 550 respectively. We use static shortest path routing as the routing protocol. The algorithm is simulated over two simple mesh network topologies as shown in Fig. 6.7. We study the performance of resource allocation algorithm in terms of system stability and fairness for flows with different lengths.



(a) Flows with the same length (b) Flows with different lengths

Fig. 6.7. Simulation Topologies.

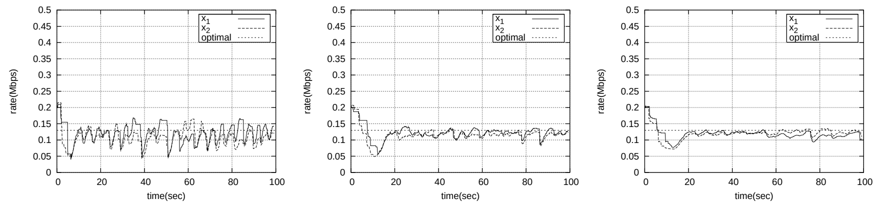
We first study the instantaneous behavior of the resource allocation algorithm and investigate the stability of the system with different settings of parameter γ and initial flow rate x_f. In this experiment, our algorithm is simulated over the topology in Fig. 6.7(a). The default parameter values are set as follows: $\gamma = 0.005$, $\beta = 10$, and $x_1(0) = x_2(0) = 200Kbps$. Fig. 6.8 plots the instantaneous throughput of the system with different initial sending rates $x_f(0)$ along with the optimal rates. The results show that the system stabilizes around the optimal rate, independent of the initial condition. The small fluctuations around the optimal value are caused by the imprecise channel measurements and the communication delay between the flow sources and the intermediate nodes where prices are generated.



(a) $x(0) = 20Kbps$ (b) $x(0) = 200Kbps$ (c) $x(0) = 500Kbps$

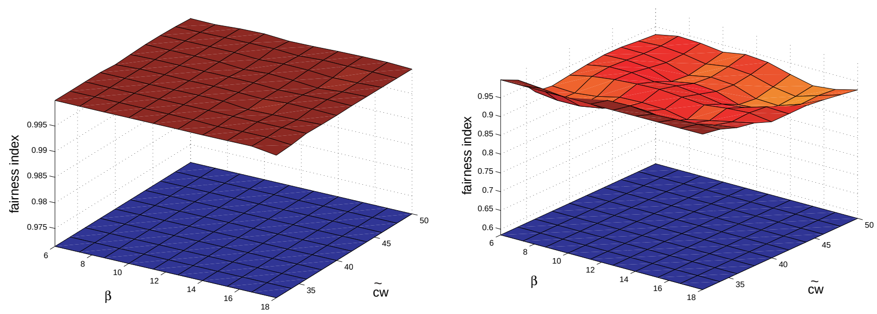
Fig. 6.8. Instantaneous Throughput with Different Initial Sending Rates.

Fig. 6.9 shows that the value of κ will affect the stability and the convergence rate of the algorithm. In particular, if κ is too large (*e.g.*, $\kappa = 0.005$), the flow rates always fluctuate. The value of κ needs to be small enough (< 0.002) to ensure the stabilization of the system. On the other hand, the algorithm will converge slower with smaller κ value.



(a) $\gamma = 0.005$ (b) $\gamma = 0.002$ (c) $\gamma = 0.001$

Fig. 6.9. Instantaneous Throughput with Different Values of γ.



(a) fairness with symmetric flow (b) fairness with asymmetric flow

Fig. 6.10. Fairness.

We now proceed to show the fairness of our price-based rate allocation. Recall that when the utility function is set to $U_f(x_f) = \log(x_f)$, the price-based rate allocation is able to achieve proportional fairness [17]. Here we present some intuitive properties of proportional fairness. (1) If flows f_1 and f_2 share the same path (uses the same amount of resources), then $x_1 = x_2$; (2) if flow f_1 uses more bottleneck resources than f_2, then $x_1 < x_2$. Specifically, if the prices of flow f_1 and f_2 are κ_1 and κ_2, then $\frac{x_1}{x_2} = \frac{\kappa_2}{\kappa_1}$. To quantitatively study the property of fairness, we define the fairness index in the single resource element case (no spatial reuse) as $I = \frac{(\sum_{f \in F}(x_f/H)^2}{|F| \times \sum_{f \in F}(x_f/H)^2}$, where H is the number of hops of the flow passes in this channel. Note that $I \in [0,1]$. Larger value of I indicates better fairness. Fig. 6.10 plots the fairness index in the simulation scenario as shown in Fig. 6.7. We compare our algorithm with TCP, which is shown to be unfair for end-to-end flows in wireless backhaul mesh network [15]. The results show that our algorithm outperforms TCP in terms of fairness, independent of the values of β and $\tilde{cw}$.

6.6 Related Work

We compare and highlight the contributions of this work in light of previous related work.

Existing research on WMN has focused on how to better utilize the wireless channel resource and enhance its performance. Proposed solutions include equipping mesh nodes with multiple radios and distributing the wireless backbone traffic over different channels, routing the traffic through different paths [18, 19], or a joint solution of these two [20, 21]. These existing approaches usually fall into two ends of the spectrum. On one end of the spectrum are the heuristic algorithms (*e.g.*, [21, 18]). Although many of such approaches are adaptive to the dynamic environments of wireless networks, they lack the theoretical foundation to analyze how well the network performs globally (*e.g.*, whether the network resource is fully utilized, whether the flows share the network in a fair fashion). On the other end of the spectrum, there are theoretical studies that formulate these network planning decisions into optimization problems (*e.g.*, [22, 23]). Yet these results usually make ideal assumptions and present centralized algorithms. None of them has realistically considered the highly dynamic and distributed nature of wireless mesh network environments. Further, these existing solutions only apply to routing and channel allocation, while our work addresses the resource allocation and rate limiting problem for wireless mesh network.

The problem of optimal and fair resource allocation has been extensively studied in the context of wireline networks, where pricing has been shown to be an effective approach (*e.g.*, [9, 24, 25]). Our approach is similar to [9, 25], which solves the resource allocation problem using a penalty-based approach. Nevertheless, the fundamental differences in contention models between multihop wireless and wireline networks deserve a fresh treatment to this topic. One of the highlights of this work is to propose a generalized theoretical framework of resource allocation that fits both wireline and multihop wireless networks. Within this framework we show that the resource model of wireline network is just a special case, in comparison with multihop wireless networks.

The problem of fair and effective resource allocation in multihop wireless networks has also been previously studied, using MAC-layer fair scheduling which targets on *single-hop* MAC layer flows [26, 27, 26, 28]. In comparison, this work studies *end-to-end* multihop flows in such networks. It can be shown that fair resource allocation among single-hop flows may not be optimal for multi-hop flows, due to the unawareness of bottlenecks and lack of coordination among upstream and downstream hops. Moreover, global optimal resource allocation among multi-hop flows can not be completely reached only by MAC-layer scheduling, which is only based on local information. In this context, the only remedial solution is to use prices as signals to coordinate global resource allocation.

This work is also related to the work of [29] and [11], in that both works explore the fundamental performance limit of a multihop wireless network in presence of interference. Yet our work is different from these works in the following aspects. First, we seek to maximize the aggregated utility, which can be a nonlinear function of flow rates. Such an objective can achieve maximum throughput under different fairness models by specifying appropriate utility functions. Second, the solution space of their approach is defined by routing [11] or joint scheduling and routing [29], while this work defines its solution space by rate control. Third, the works of [29, 11] aim at deriving the limit of optimal throughput by centralized algorithms. This work focuses on how to achieve such a limit, by presenting a decentralized algorithm and a distributed implementation.

The work of [30] provides an intuitive solution to improve TCP fairness via neighborhood RED. Our work can be regarded as its theoretical interpretation. Our early work in [16] also presents a resource allocation algorithm for multihop wireless networks. Compared to [16], this chapter presents a more precise and general resource model from the view of independent set polytope. Moreover, in terms of resource allocation algorithm, [16] uses a dual approach which provides an exact solution to the original resource allocation problem directly. In contrast, the resource allocation framework presented in this paper uses a primal approach that solves the relaxation of the original problem. We argue that this approach may be more suitable for real deployment in multihop wireless mesh networks, as the prices can be generated directly from channel conditions.

A collection of papers have studied the use of price in the context of wireless networks (*e.g.*, [31, 32]). In these papers, pricing has been used as a mechanism for optimal distributed power control. In addition, Liao *et al.* [33] use prices to provide incentives for service allocation in wireless LANs. The work of [34, 35] also uses prices as incentives to encourage packet relays in multihop wireless networks. Our work is different from these works in that we apply pricing to regulate resource usage rather than providing incentives.

6.7 Summary

This chapter targets on the resource allocation problem in wireless mesh network. What we desire is a generalized theoretical framework, which can effectively capture the common nature of these problems, *i.e.*, they can all be categorized as constrained non-linear optimization problem. Applying this generalized framework to the setting of multihop wireless mesh backbone network, we find out that a resource element is not a wireless link, but a facet of the polytope determined by independent sets in the conflict graph of the wireless network. Through this finding, we are able to outline the solution space of the resource allocation problem in wireless mesh network, and derive the corresponding decentralized algorithm. The same framework also provides theoretical evidence to help judge the feasibility of existing solutions. We reveal that the fundamental problem of TCP unfairness in multihop wireless networks lies in its incorrect congestion signal (*i.e.*, price). Our work also offers theoretical validation to recent-proposed solutions, such as neighborhood RED [30], IFA [15].

References

1. “Mesh networks inc.,” http://www.meshnetworks.com.
2. “Radiant networks,” http://www.radiantnetworks.com.
3. “Seattle wireless,” http://www.seattlewireless.net.

4. "MIT roofnet," http://www.pdos.lcs.mit.edu/roofnet/.
5. "Chaska wireless solutions," http://www.chaska.net/.
6. R. Karrer, A. Sabharwal, and E. Knightly, "Enabling large-scale wireless broadband: the case for taps," *ACM SIGCOMM Comput. Commun. Rev.*, vol. 34, no. 1, pp. 27–32, 2004.
7. J. Bicket, D. Aguayo, S. Biswas, and R. Morris, "Architecture and evaluation of an unplanned 802.11b mesh network," in *Proc. of AMC MobiCom*, 2005.
8. F. P. Kelly, "Charging and rate control for elastic traffic," *European Trans. on Telecommunications*, vol. 8, pp. 33–37, 1997.
9. F. P. Kelly, A. K. Maulloo, and D. K. H. Tan, "Rate control in communication networks: Shadow prices, proportional fairness and stability," *Journal of the Operational Research Society*, vol. 49, pp. 237–252, 1998.
10. P. Gupta and P.R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, pp. 388–404, 2000.
11. K. Jain, J. Padhye, V. Padmanabhan, and L. Qiu, "Impact on interference on multi-hop wireless network performance," in *Proc. of ACM MobiCom*, 2003.
12. M. de Berg, M. van Kreveld, M. Overmars, and O. Schwarzkopf, "Computational geometry: algorithms and applications," *Springer*, 2000.
13. J. G. Augustson and J. Minker, "An analysis of some graph theoretical cluster techniques," *Journal of ACM*, vol. 17, no. 4, pp. 571–586, 1970.
14. K. Jain, J. Padhye, V. Padmanabhan, and L. Qiu, "Impact on interference on multi-hop wireless network performance," in *Proc. of ACM Mobicom*, 2003.
15. V. Gambiroza, B. Sadeghi, and E. W. Knightly, "End-to-end performance and fairness in multihop wireless backhaul networks," in *Proc. of ACM MobiCom*, 2004.
16. Y. Xue, B. Li, and K. Nahrstedt, "Optimal resource allocation in wireless ad hoc networks: A price-based approach," *IEEE Transactions on Mobile Computing*, vol. 5, no. 4, pp. 347–364, April 2006.
17. S. Kunniyur and R. Srikant, "End-to-end congestion control: utility functions, random losses and ECN marks," in *Proc. of IEEE INFOCOM*, 2000.
18. R. Draves, J. Padhye, and B. Zill, "Routing in multi-radio, multi-hop wireless mesh networks," in *Proc. of ACM Mobicom*, 2004.
19. Y. Yuan, H. Yang, S. H. Y. Wong, S. Lu, and W. Arbaugh, "Romer: resilient opportunistic mesh routing for wireless mesh networks," in *Proc. of IEEE WiMesh*, 2005.
20. A. Raniwala, K. Gopalan, and T. Chiueh, "Centralized channel assignment and routing algorithms for multi-channel wireless mesh networks," *Mobile Computing and Communications Review*, vol. 8, no. 2, pp. 50–65, 2004.
21. A. Raniwala and T. Chiueh, "Architecture and algorithms for an IEEE 802.11-based multi-channel wireless mesh network," in *Proc. of IEEE INFOCOM*, 2005.
22. M. Alicherry, R. Bhatia, and L. Li, "Joint channel assignment and routing for throughput optimization in multi-radio wireless mesh networks," in *Proc. of ACM MobiCom*, 2005.
23. M. Kodialam and T. Nandagopal, "Characterizing the capacity region in multi-radio multi-channel wireless mesh networks," in *Proc. of IEEE WiMesh*, 2005.
24. S. H. Low and D. E. Lapsley, "Optimization flow control, I: basic algorithm and convergence," *IEEE/ACM Trans. on Networking*, vol. 7, no. 6, pp. 861–874, 1999.
25. S. Kunniyur and R. Srikant, "End-to-end congestion control: utility functions, random losses and ECN marks," in *Proc. of IEEE INFOCOM*, 2000.
26. H. Luo, S. Lu, and V. Bharghavan, "A new model for packet scheduling in multihop wireless networks," in *Proc. of ACM Mobicom*, 2000.
27. L. Tassiulas and S. Sarkar, "Maxmin fair scheduling in wireless networks," in *Proc. of IEEE INFOCOM*, 2002.

28. Y. Liu and E. Knightly, "Opportunistic fair scheduling over multiple wireless channels," in *Proc. of IEEE INFOCOM*, 2003.
29. M. Kodialam and T. Nandagopal, "Characterizing the achievable rates in multihop wireless networks," in *Proc. of ACM Mobicom*, 2003.
30. K. Xu, M. Gerla, L. Qi and Y. Shu, "Enhancing TCP fairness in ad hoc wireless networks using neighborhood RED," in *Proc. of ACM Mobicom*, 2003.
31. T. M. Heikkinen, "On congestion pricing in a wireless network," *Wireless Networks*, vol. 8, no. 4, pp. 347–354, 2002.
32. D. Julian, M. Chiang, D. O'Neill, and S. Boyd, "QoS and fairness constrained convex optimization of resource allocation for wireless cellular and ad hoc networks," in *Proc. of IEEE INFOCOM*, 2002.
33. R. Liao, R. Wouhaybi, and A. Campbell, "Incentive engineering in wireless LAN based access networks," in *Proc. of IEEE ICNP*, 2002.
34. Y. Qiu and P. Marbach, "Bandwidth allocation in ad-hoc networks: A price-based approach," in *Proc. of IEEE INFOCOM*, 2003.
35. L. Buttyan and J. P. Hubaux, "Stimulating cooperation in self-organizing mobile ad hoc networks," *ACM/Kluwer Mobile Networks and Applications*, vol. 8, no. 5, October 2003.